Noise Figures of Radio Receivers*

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Summary—**A rigorous definition of the noise figure of radio receivers is given in this paper. The definition is not limited to high-gain receivers, but can be applied to four-terminal networks in general. An analysis is made of the relationship between the noise figure of the receiver as a whole and the noise figures of its components. Mismatch relations between the components of the receiver and methods of measurements of noise figures are discussed briefly.**

Introduction

The importance of noise originating in a radio receiver has increased as shorter and shorter wavelengths have come into practical usage. Many papers on the subject, notably those by Llewellyn[1] and Jansky,[2] have appeared since the writer, in 1928, showed experimentally[3] that thermal-agitation noise (Johnson noise) determined the absolute sensitivity of short-wave radio receivers. Early in 1942 North[4] suggested the adoption of a standard for the absolute sensitivity of radio receivers which differed by a factor of 2 from the standard used by us at that time. We adopted his standard, since ours was somewhat limited in that it was based on matched impedances in the input circuit of the receiver.

In this paper a more rigorous definition of the standard of absolute sensitivity, the so-called noise figure, of a radio receiver is suggested. The definition is not limited to high-gain receivers, but can be applied to four-terminal networks in general. It also makes it possible by a simple analysis to give the relationship between the noise figure of the receiver as a whole and the noise figures of its components. In the case of a double-detection receiver these components may be a high-frequency amplifier, a frequency converter, and an intermediate-frequency amplifier. The paper also gives a brief description of methods of measurements of noise figures.

The four-terminal network whose noise figure is to be defined is shown schematically in Fig. 1. A signal generator is connected to its input terminals and an output circuit is also indicated. The input and output impedances of the network may have reactive components and they may be matched or mismatched to the generator and the output circuit, respectively. The four-terminal network may be, for instance, an amplifier, a converter, an attenuator, or a simple transformer. The presence of the signal generator is required for the definition that follows, but the attenuator in the signal generator and the output circuit toward the right are shown only to illustrate measurements of noise figures and gains.

The noise figure will be defined in terms of available signal power, available noise power, gain, and effective bandwidth. The definitions of these terms will be given and discussed next.

Available Signal Power

A generator with an internal impedance R_0 ohms and electromotive force E volts delivers $E^2R_1/(R_0+R_1)^2$ watts into a resistance R_1 ohms. This power is maximum and equal to $E^2/4R_0$ when the output circuit is matched to the generator impedance, that is when $R_1=R_0$. $E^2/4R_0$ is hereafter called the available power of the generator, and it is, by definition, independent of the impedance of the circuit to which it is connected. The output power is smaller than the available power when R_1 is unlike R_0, since there is a mismatch loss. In amplifier input circuits a mismatch condition may be beneficial[5] due to the fact that it may decrease the output noise more than the output signal. It is the presence of such mismatch conditions in amplifier input circuits that makes it desirable to use the term available power in this paper. The symbol S_g will be used for the available signal power at the output terminals of the signal generator shown in Fig. 1. S_g is here equal to V^2/RA watts where V is the voltage across the input terminals

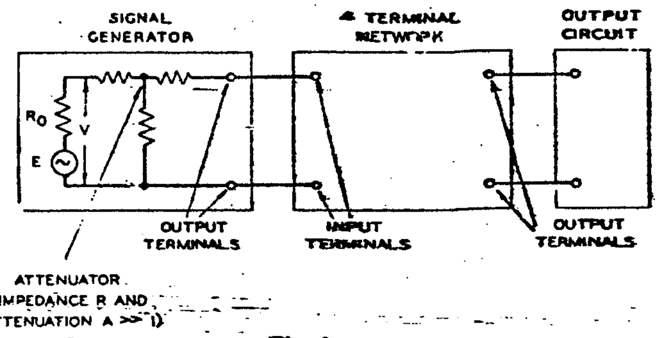

Fig. 1

of the attenuator, R the characteristic impedance of the attenuator, and A the nominal attenuation (A is assumed large).

The output terminals of any network may be considered as being the output terminals of a signal generator. The symbol S will be used for the available signal power at the output terminals of the four-terminal network shown in Fig. 1.

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[1] F. B. Llewellyn, "A study of noise in vacuum tubes and attached circuits," Proc. I.R.E., vol. 18, pp. 243–266; February, 1930.

[2] K. G. Jansky, "Minimum noise levels obtained on short-wave radio receiving systems," Proc. I.R.E., vol. 25, pp. 1517–1531; December, 1937.

[3] Unpublished report.

[4] D. O. North, "The absolute sensitivity of radio receivers," *RCA Rev.*, vol. 6, pp. 332–344; January, 1942. The reader is referred to this paper for reference to other papers on this subject.

[5] That such an improvement might be possible was first discussed in detail by F. B. Llewellyn in his paper "A rapid method of estimating the signal-to-noise ratio of a high gain receiver." Proc. I.R.E., vol. 19, pp. 416–421; March, 1931.

Gain

The gain of the network is defined as the ratio of the available signal power at the output terminals of the network to the available signal power at the output terminals of the signal generator. Hence

$$G = S/S_g. \quad (1)$$

This is an unusual definition of gain since the gain of an amplifier is generally defined as the ratio of its output and input powers. This new definition is introduced here for the same reason that made it desirable to use the term available power. Note that while the gain is independent of the impedance which the output circuit presents to the network, it does depend on the impedance of the signal generator.

The four-terminal network has generally some kind of band-pass characteristic. The gain G is defined as that at the mid-band frequency.

Available Noise Power

As in the case of signal power, the available noise power between two terminals is defined as the noise power which would be absorbed by a matched output circuit.

The symbol N will be used for the available noise power at the output terminals of the network. This power is due to all the noise sources in the network itself and the Johnson-noise sources in the signal generator, but noise sources in the output circuit toward the right in Fig. 1 are not included.

The Johnson-noise power available from a resistance will be discussed now. Any resistance, R ohms, acts as a Johnson-noise generator with a mean-square electromotive force equal to $4KTRdf$. K is Boltzmann's constant $= 1.38 \times 10^{-23}$, T is the absolute temperature of the resistance, and df is the bandwidth. The available Johnson-noise power is then

$$4KTRdf/4R = KTdf \text{ watts} \quad (2)$$

and this is the noise power available over the band df at the output terminals of the signal generator in Fig. 1. It is, in fact, the available noise power between any two terminals of a passive network when all its parts have the same temperature T.

Effective Bandwidth

The contribution to the available output noise by the Johnson-noise sources in the signal generator is readily calculated for an ideal or square-top band-pass characteristic and it is $GKTB$ where B is the bandwidth in cycles per second. In practice, however, the band is not flat; i.e., the gain over the band is not constant but varies with the frequency. In this case, the total contribution is $\int G_f KTdf$ where G_f is the gain at the frequency f. The effective bandwidth B of the network is defined as the bandwidth of an ideal band-pass network with gain G that gives this contribution to the noise output. Therefore,

$$GKTB = \int G_f KTdf$$

or

$$B = \frac{1}{G}\int G_f df. \quad (3)$$

Noise Figures

The noise figure of the network in Fig. 1 will now be defined in terms of S_g, S, KTB, and N.

It is important to have the highest possible signal-to-noise ratio at the output terminals of the network. The maximum value of this ratio would be as high as the available-signal-to-available-noise ratio at the signal generator terminals if there were absolutely no noise sources present in the network. Simple lossless transformers or filters are examples of networks with no noise sources. In general, however, noise sources are present and these noise sources reduce the available signal-to-noise ratio at the output terminals of the network. The noise figure F of the network[6] is defined as the ratio of the available signal-to-noise ratio at the signal-generator terminals to the available signal-to-noise ratio at its output terminals.[7] Thus

$$F = (S_g/KTB)/(S/N) = (S_g/KTB)(N/S) \quad (4)$$

and since

$$G = S/S_g$$

$$F = (1/G)(N/KTB). \quad (5)$$

Solving for N gives the following expression for the available noise output:

$$N = FGKTB \text{ watts}. \quad (6)$$

This noise output includes the contribution made by the Johnson-noise source in the signal generator. This contribution is $GKTB$. The available output noise due only to noise sources in the network is, therefore,

$$(F-1)GKTB \text{ watts}. \quad (7)$$

All the terms in (4), (5), (6), and (7) have been defined, but a value for the temperature T of the generator terminal impedance must still be chosen before the noise figure is definite. It is suggested that the noise figure be defined for a temperature of 290 degrees Kelvin (63 degrees Fahrenheit). Then

$$KT = 1.38 \times 10^{-23} \times 290$$
$$= 4 \times 10^{-21} \text{ watts per cycle bandwidth.}$$

The relationship between the noise figure and the degree of mismatch that exists between the network and its output and input circuits is important. Definition (4) shows clearly that the output circuit and hence its coupling with the network has no effect on the value of the noise figure. However, it also shows that the noise figure does depend on the degree of mismatch between the generator and the network since both S and N will vary with the magnitude of this mismatch.

[6] We have, until now, used the symbol $\overline{NF}$ for the noise figure, but we shall use, hereafter, the symbol F suggested by Dr. S. Roberts.
[7] Because of this definition, the noise figure has also been called "excess noise ratio."

Measurement of the Noise Figure

Although a detailed description of noise-figure-measuring equipment will not be given in this paper, it is believed worth while to outline a method of such measurements.

It is not difficult to measure the noise figure F when the gain of the network is so large that a noise-power reading can be obtained by means of a thermocouple connected between the output terminals of the network (Fig. 1). The measurement procedure is then simply to adjust the attenuation A of the signal attenuator until the output reading is double that due to noise only which is obtained with the signal generator turned off. S is then equal to N and definition (4) gives

$$F = S_s/KTB = V^2/4RAB \times 10^{19}. \qquad (8)$$

The effective band B is calculated from a gain-versus-frequency curve. The voltage V across the input terminals of the signal attenuator may be measured by means of thermocouples, tube voltmeters, thermistors, etc., and by cross-checking such different equipment the value of V can be obtained with adequate accuracy even in the centimeter-wavelength range. It is more difficult to obtain an accurate value of the attenuation A because of its large magnitude. For a short-wave receiver for which F may be as small as 3, formula (8) gives $A = 5.2 \times 10^{11}$ for $R = 80$ ohms, $V = 1$ volt, and $B = 20{,}000$ cycles. Only very careful work will give satisfactory data with such magnitudes of attenuation. Very thorough shielding of the signal generator is one important requirement.

The noise figure of a network made up of nondissipative elements is unity since it contains no noise sources (expression (7) is equal to zero). The losses in simple transformers and filters are generally sufficiently low to come under this classification. The noise figure of an attenuator at 63 degrees Fahrenheit temperature is by (5) equal to its attenuation when it is matched to the signal generator since under these conditions $N = KTB$ and attenuation $= 1/G$. A network made up of a transmitting and a receiving antenna is equivalent to an attenuator with an attenuation A equal to the ratio of transmitted to received power. Assuming no static or star noise and no circuit losses in the receiving antenna, its noise figure is, by (5), $F = A(N/KTB)$. If T_r is the absolute temperature of the radiation resistance of the receiving antenna, $N = KT_rB$. Hence $F = A(T_r/T) = A(T_r/290)$. The value of T_r is not definitely known, but $T_r = T$ is believed to be a good approximation for antennas whose radiation strikes the earth.[8]

Noise Figures for Two Networks in Cascade

If the gain of the network shown in Fig. 1 is not large, an amplifier following the network is required to obtain a noise-output reading. For this case a noise-figure analysis of two networks in cascade is required. Also from a design point of view it is important to know the relationship between the noise figure of a whole receiver and the noise figures of its components since it indicates the component on which efforts for improvement are worth while.

The two networks are shown schematically in Fig. 2. We are also considering here the general case where the two networks, the generator, and the output circuit may be either matched or mismatched. The definitions given for a single network will now be applied to the network ab made up of the two networks a and b in cascade and to the individual networks a and b.

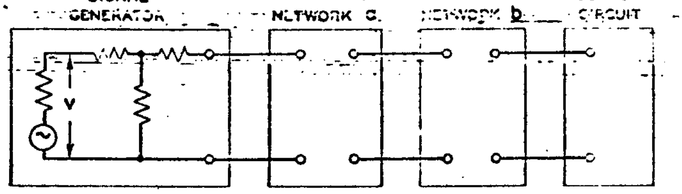

Fig. 2

Expression (6) gives for the available output noise at the output terminals of network b

$$N'_{ab} = F_a G_{ab} KTB_{ab} \text{ watts.} \qquad (9)$$

To simplify the analysis, it will be assumed that the two networks have the same ideal or square-top band-pass characteristics ($B_a = B_b = B$). The equivalent band B_{ab} is then equal to B. The total gain G_{ab} is by the definition of gain (1) equal to G_aG_b. Then

$$N_{ab} = F_{ab} G_a G_b KTB \text{ watts.} \qquad (10)$$

A new expression for this noise power may be derived by applying expression (6) to network a and expression (7) to network b. Applying (6) to network a, the available noise power at its output terminals is

$$N_a = F_a G_a KTB \text{ watts.} \qquad (11)$$

Multiplying this power by G_b gives then the following expression for the available noise power at the output terminals of network b due to the noise sources in network a and the Johnson-noise sources in the signal generator

$$F_a G_a G_b KTB \text{ watts.} \qquad (12)$$

Expression (7) applied to network b gives the following expression for the available noise power at the output terminals of network b due to noise sources in network b only,

$$(F_b - 1) G_b KTB \text{ watts.} \qquad (13)$$

The total available noise power N_{ab} at the output terminals of network b is the sum of the noise powers given by (12) and (13), hence,

$$N_{ab} = F_a G_a G_b KTB + (F_b - 1) G_b KTB$$
$$= \left(F_a + \frac{F_b - 1}{G_a}\right) G_a G_b KTB. \qquad (14)$$

Comparing this expression with (10) gives the following simple relationship between the noise figures of the two networks

$$\cancel{F_{ab} = (F_a + F_b) - 1/G_a}. \qquad (15)$$

$F_{ab} = F_a + \frac{F_b - 1}{G_a}$

[8] For further information on this subject the reader is referred to a paper by R. E. Burgess, "Noise in receiving aerial systems," *Proc. Phys. Soc.*, vol. 53, pp. 293–304; May, 1941.

This relationship is valid for any distribution of noise power throughout the bands of the two networks. The assumptions made in regard to the band-pass characteristics could be made less severe for uniform noise-power distribution. Although they do not seriously limit the usefulness of relationship (15) for practical cases, it is recommended that both the effect of nonuniform noise distribution and the effect of unequal and nonideal bands be studied to make sure whether it is necessary to use correction factors for the different terms of (15).

The rather complicated matter of the effects of nonuniform noise distribution and nonideal band characteristics may be clarified somewhat by pointing out that the relationship (15) may always be applied to an element of band df at a frequency f in the band of the networks. It usually will be found that the noise figure corresponding to an element of band varies somewhat across the band of an actual network.

The noise figures F_a and F_b in (15) will be discussed next. Network b has no effect on the noise figure F_a of network a. This follows from the discussion of a single network. That discussion also pointed out that network a does affect the noise figure F_b of network b. Therefore, if F_b is measured separately by a signal generator, as shown in Fig. 1, then this signal generator must have a terminal impedance which is identical to the output impedance between the output terminals of network a.

Mismatch Relations for Two Networks

The reader is referred to a paper by Burgess[8] and a more recent paper by Herold[9] for detailed discussions of the advantage of mismatch relations, and only a brief discussion will be given here.

The over-all noise figure for two networks has a minimum value when the degree of mismatch between them is made identical to the mismatch which gives the lowest noise figure for the second network when it is connected directly to a signal generator. Offhand, this is not evident, but an analysis of the matching condition between a signal generator and a network shows that the optimum matching condition is independent of any noise sources in the signal generator. For the lowest over-all noise figure, the optimum matching condition between the signal generator and network a does, on the other hand, depend on both networks. When network a is a low-gain converter and network b an intermediate-frequency amplifier, the highest possible gain G_a in the first network, which is obtained when it is matched to the signal generator, will in general give the lowest over-all noise figure.

Noise Figures for Several Networks in Cascade

The analysis for two networks may be easily extended to more than two networks. For example, if three networks are considered, (15) gives

$$F_{abc} = F_{ab} + (F_c - 1)/G_{ab} = F_a + (F_b - 1)/G_a + (F_c - 1)/G_aG_b. \qquad (16)$$

In most receivers the gains of the amplification stages are such that the noise figures of only the first two stages must be considered.

Measurement of the Noise Figures of Two Networks

It may be desirable to determine F_a by indirect measurements, particularly if G_a is low. This may be done as follows. The noise figures F_{ab} and F_b may be measured by the method described for a single network. The gain G_a may be obtained, by means of signal generators, as the increase in available signal power required to give a certain signal output reading when network a is left out. The noise figure F_a may then be calculated by means of (15).

A second method of noise-figure measurement includes the measurement of the ratio of the output noise of network b with network a in normal operation to that with network a passive. This ratio is called the Y figure and is especially useful in converter measurements. Network a is said to be passive if its available output noise is only KTB watts. In measuring Y it is usually most convenient to replace the output impedance of network a with an equal passive impedance at room temperature.

An expression for F_a in terms of Y, F_b, and G_a will be developed next. By definition,

$$Y = N_{ab}/N_b. \qquad (17)$$

Formula (5) gives

$$F_b = (1/G_b)(N_b/KTB_b) \qquad (18)$$

and

$$F_{ab} = (1/G_aG_b)(N_{ab}/KTB_{ab}). \qquad (19)$$

The above three formulas give

$$F_{ab} = (F_bY/G_a)(B_b/B_{ab}). \qquad (20)$$

It will also here be assumed that the two networks have the same ideal or square-top band-pass characteristic. Then

$$F_{ab} = F_bY/G_a. \qquad (21)$$

Formulas (15) and (21) give

$$F_a = [F_b(Y - 1) + 1]/G_a. \qquad (22)$$

Formula (22) is often simpler to use for the determination of F_a than relationship (15) because it is easier to measure Y than F_{ab}. Note also that F_{ab} may be determined experimentally from the convenient relation given by (21).

Conclusion

All signal and noise powers are in watts. It is confusing to use the decibel scale when noise powers are added, and it has not, therefore, been used in this paper.

In concluding, it is hoped that the definitions and symbols suggested will come into general usage. It should be pointed out that the paper is the result of a great many discussions during the past two years with scientific workers both inside and outside the Bell Telephone Laboratories.

[9] E. W. Herold, "An analysis of the signal-to-noise ratio of ultra-high-frequency receivers," *RCA Rev.*, vol. 6, pp. 302–332, January, 1942.